

FORMULES DE TRIGONOMETRIE CIRCULAIRE

Pour tous les réels a, b, p, q et x :

$$\begin{aligned}\cos^2 x + \sin^2 x &= 1 \\ 1 + \tan^2 x &= \frac{1}{\cos^2 x}\end{aligned}$$

Formules d'addition

$$\begin{aligned}\cos(a + b) &= \cos a \cos b - \sin a \sin b \\ \sin(a + b) &= \sin a \cos b + \sin b \cos a \\ \tan(a + b) &= \frac{\tan a + \tan b}{1 - \tan a \tan b}\end{aligned}$$

Formules de linéarisation

$$\begin{aligned}\cos a \cos b &= \frac{1}{2}[\cos(a + b) + \cos(a - b)] \\ \sin a \cos b &= \frac{1}{2}[\sin(a + b) + \sin(a - b)] \\ \sin a \sin b &= \frac{1}{2}[\cos(a - b) - \cos(a + b)]\end{aligned}$$

Formules de factorisation

$$\begin{aligned}\cos p + \cos q &= 2 \cos \frac{p + q}{2} \cos \frac{p - q}{2} \\ \cos p - \cos q &= -2 \sin \frac{p + q}{2} \sin \frac{p - q}{2} \\ \sin p + \sin q &= 2 \sin \frac{p + q}{2} \cos \frac{p - q}{2} \\ \sin p - \sin q &= 2 \sin \frac{p - q}{2} \cos \frac{p + q}{2}\end{aligned}$$

Formules de duplication

$$\begin{aligned}\cos(2a) &= \cos^2 a - \sin^2 a = 2 \cos^2 a - 1 = 1 - 2 \sin^2 a \\ \sin(2a) &= 2 \sin a \cos a \\ \tan(2a) &= \frac{2 \tan a}{1 - \tan^2 a}\end{aligned}$$

Pour $t = \tan\left(\frac{x}{2}\right)$:

$$\begin{aligned}\cos x &= \frac{1 - t^2}{1 + t^2} \\ \sin x &= \frac{2t}{1 + t^2} \\ \tan x &= \frac{2t}{1 - t^2}\end{aligned}$$

Pour $n \in \mathbb{Z}$:

$$\begin{aligned}\cos(x + n\pi) &= (-1)^n \cos x \\ \sin(x + n\pi) &= (-1)^n \sin x\end{aligned}$$